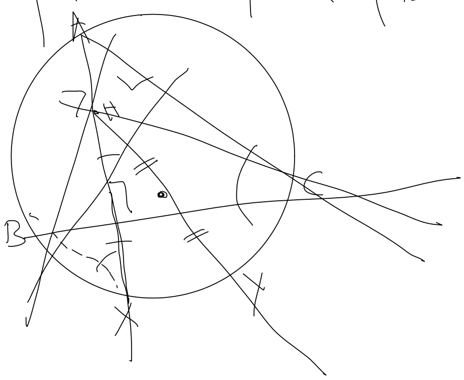
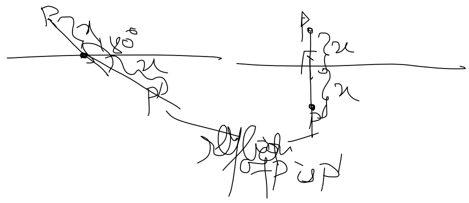
Lemma – Let H be the orthocentre of ΔABC. Let X be the reflection of H over $\overline{BC}$ and Y be the reflection of H over midpoint of $\overline{BC}$

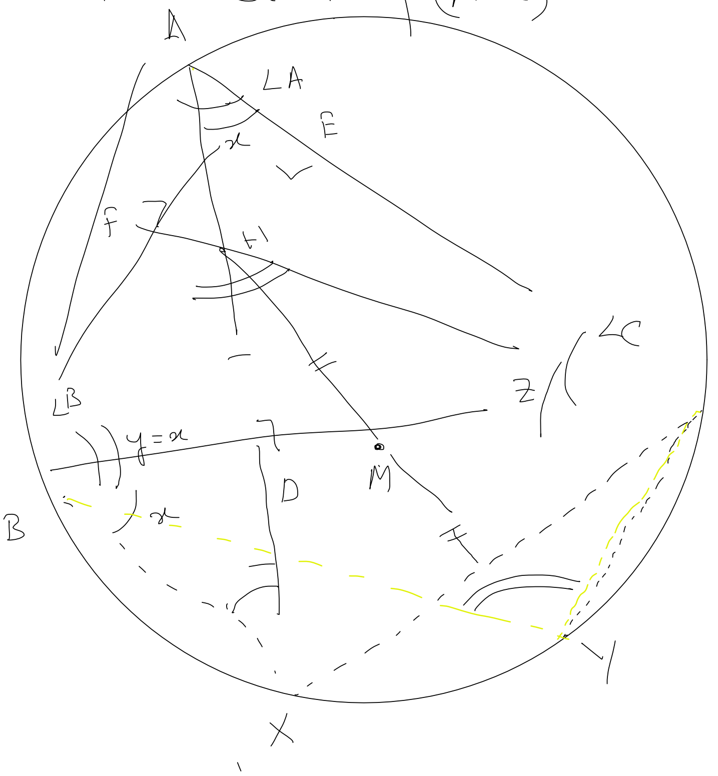




(a) Show that X lies on circumcircle of ΔABC (also written as (ABC))

(b) Show that $\overline{AY}$ is a diameter of (ABC)

Ans:-



$BM = MC$,
$HM = MY$
$\Leftrightarrow$ BHCY parallelogram

In ΔBDH and ΔBDX,
$\overline{BD}$ is common,
$\angle BDH = \angle BDX = 90^\circ$

In ΔADC,
$x = 90^\circ - \angle C$

In ΔBFC,
$y = 90^\circ - \angle C = x$

In ABXC,
$x = \angle CBX$

$\Rightarrow \Delta BDH \cong \Delta BDX$
$\Rightarrow HD = XD$

In ΔBMH and ΔMCY, $MC = BM$ and $\angle BMH = \angle CMY$

$z = 90^\circ - \angle B$, $y = 90^\circ - \angle C$

$\angle BHC = 180^\circ - y - z = 90^\circ - y + 90^\circ - z = \angle B + \angle C$

$\Rightarrow$ BHCY is a parallelogram $\Rightarrow \angle BYC = \angle BHC$

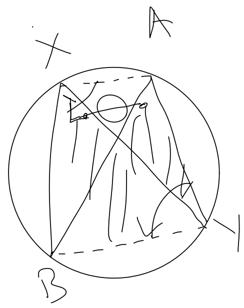
$\Rightarrow \angle BYC = \angle B + \angle C = 180^\circ - \angle A$

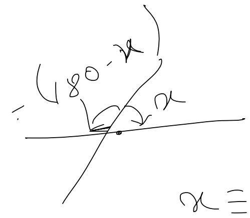
$\Rightarrow$ ABYC is cyclic quadrilateral

$\Rightarrow$ Y is in circumference.

Theorem:- (Cyclic Quadrilateral with Directed Angles)

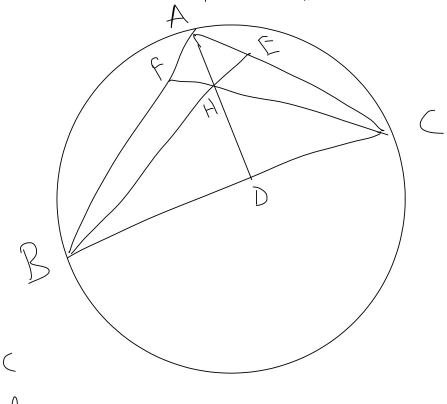
Points A, B, X, Y lie on a circle iff $\measuredangle AXB = \measuredangle AYB$





$x \equiv -(180-x)$

Q) Let H be the orthocentre of ΔABC. Using directed angles show that AEHF, BFHD, CDHE, CFDA and ADEB are cyclic.



Ans:- $\measuredangle AEH = \measuredangle AFH = 90^\circ \Rightarrow$ AEHF is cyclic

BFHD, CDHE are similarly cyclic

$\measuredangle CDA = \measuredangle CFA = 90^\circ \Rightarrow$ CFDA is cyclic

$\measuredangle AEB = \measuredangle ADB = 90^\circ \Rightarrow$ ADEB is cyclic

HomeWork:-

Q) Show that for any distinct points ABCD, we have,

$$\measuredangle ABC + \measuredangle BCD + \measuredangle CDA + \measuredangle DAB = 0$$